# **BRIEF COMMUNICATION**

## A METHOD FOR DETERMINING FRICTIONAL PRESSURE LOSSES IN TWO PHASE FLOW

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An equation is presented for the extraction of frictional pressure loss from the total pressure drop in churn-turbulent bubble flow in a vertical pipe. This method accounts for the dependence of voidage on frictional loss, which may be a significant factor at high liquid flowrates.

Let the absolute pressure at two stations in a vertical pipe,  $x_1$  and  $x_2$ , be  $P_1$  and  $P_2$ . As an accurate approximation to the Zuber & Findlay (1965) drift-flux model the gas void fraction at some height in the pipe is

$$\epsilon = \epsilon_0 P_0 / P \tag{1}$$

where P is pressure and  $P_0$  is atmospheric pressure, and where

$$\epsilon_0 = W_{G0} / \{ C_0 (W_c + W_L) + V_V \}$$
<sup>[2]</sup>

where  $W_L$  is the liquid superficial velocity in the pipe section,  $W'_G$  is the gas superficial velocity evaluated at the median pressure in the section and  $W_{G0}$  is the gas superficial velocity evaluated at atmospheric pressure,  $P_0$ . The constants  $C_0$  and  $V_V$  are discussed in detail by Zuber & Findlay (1965). Neglecting acceleration effects and the head due to the gas phase, by combining [1] and [2] with the total pressure differential equation, and integrating

$$K(P_2 - P_1) + \epsilon_0 P_0 K^2 \ln \frac{P_2 - P_0 \epsilon_0 K}{P_1 - P_0 \epsilon_0 K} + \rho g(x_2 - x_1) = 0$$
[3]

where  $\rho$  is the liquid density, g is the acceleration due to gravity, and  $K = \rho g/(\rho g + D)$ : D is the frictional loss per unit length of pipe in the test section. But for the fact that K appears in the logarithmic term, [3] may be solved as a quadratic in K. In practice it is possible to substitute K = 1 in the logarithmic term as a first approximation, and proceed with solution. A further interaction will suffice to yield an accurate value of K, and hence D may be computed readily.

The accuracy of [3] rests upon the accuracy of the constants  $C_0$  and  $V_V$  used to predict the value of  $\epsilon_0$ . The drift velocity,  $V_V$ , deviates little from a value of 0.25 m/s in bubble

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flow, and  $C_0$  will remain within 10% of a value of 1.2 (Govier & Aziz 1972; Nassos & Bankoff 1967; Clark & Flemmer 1984). Use of an average gas superficial velocity,  $W_G$ , in the denominator of [2] leads to insignificant error in the final value of K, since the gas superficial velocity is much smaller than the liquid superficial velocity in the bubble flow regime.

Experimental verification of the method was carried out using a 5 m test section of 50 mm dia. pipe. The frictional pressure loss found using [3] with  $C_0 = 1.16$  and  $V_{\nu} = 0.25$  m/s (Clark & Flemmer 1983) agreed closely with the true frictional pressure loss, found from the difference between the total and hydrostatic heads across the section (figure 1). Pressures at the two stations were measured using pressure transducers and the hydrostatic head evaluated by isolating the flowing mixture in the section with quick-closing valves (Hewitt 1978).



Figure 1. Comparison of frictional pressure loss extracted using [3] with experimental pressure loss found using the quick-closing valve method.

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